NAG C Library Function Document

nag_tsa_diff (g13aac)

1 Purpose

nag_tsa_diff (g13aac) carries out non-seasonal and seasonal differencing on a time series. Information which allows the original series to be reconstituted from the differenced series is also produced. This information is required in time series forecasting.

2 Specification

```
void nag_tsa_diff (const double x[], Integer nx, Integer d, Integer ds, Integer s,
     double xd[], Integer *nxd, NagError *fail)
```

3 Description

Let $\nabla^d \nabla_s^D x_i$ be the ith value of a time series x_i, for $i = 1, 2, \ldots, n$ after non-seasonal differencing of order d and seasonal differencing of order D (with period or seasonality s). In general,

$$\begin{aligned} \nabla^d \nabla_s^D x_i &= \nabla^{d-1} \nabla_s^D x_{i+1} - \nabla^{d-1} \nabla_s^D x_i \qquad d > 0 \\ \nabla^d \nabla_s^D x_i &= \nabla^d \nabla_s^{D-1} x_{i+s} - \nabla^d \nabla_s^{D-1} x_i \qquad D > 0 \end{aligned}$$

Non-seasonal differencing up to the required order d is obtained using

$$\begin{aligned} \nabla^1 x_i &= x_{i+1} - x_i && \text{for } i = 1, 2, \ldots, (n-1) \\ \nabla^2 x_i &= \nabla^1 x_{i+1} - \nabla^1 x_i && \text{for } i = 1, 2, \ldots, (n-2) \\ &\vdots \\ \nabla^d x_i &= \nabla^{d-1} x_{i+1} - \nabla^{d-1} x_i && \text{for } i = 1, 2, \ldots, (n-d) \end{aligned}$$

Seasonal differencing up to the required order D is then obtained using

$$\begin{aligned} \nabla^d \nabla_s^1 x_i &= \nabla^d x_{i+s} - \nabla^d x_i && \text{for } i = 1, 2, \ldots, (n-d-s) \\ \nabla^d \nabla_s^2 x_i &= \nabla^d \nabla_s^1 x_{i+s} - \nabla^d \nabla_s^1 x_i && \text{for } i = 1, 2, \ldots, (n-d-2s) \\ &\vdots \\ \nabla^d \nabla_s^D x_i &= \nabla^d \nabla_s^{D+1} x_{i+s} - \nabla^d \nabla_s^{D+1} x_i && \text{for } i = 1, 2, \ldots, (n-d-D \times s) \end{aligned}$$

Mathematically, the sequence in which the differencing operations are performed does not affect the final resulting series of $m = n - d - D \times s$ values.

4 References

None.

5 Parameters

1: **x**[**nx**] – const double *Input*

On entry: the undifferenced time series, x_i, for $i = 1, 2, \ldots, n$.

2: **nx** – Integer *Input*

On entry: the number of values, n, in the undifferenced time series.

Constraint: **nx** $>$ **d** $+$ (**ds** $\times$ **s**).

3: **d** – Integer *Input*

On entry: the order of non-seasonal differencing, d.

Constraint: $\mathbf{d} \geq 0$.

4: **ds** – Integer *Input*

On entry: the order of seasonal differencing, D.

Constraint: $\mathbf{ds} \geq 0$.

5: **s** – Integer *Input*

On entry: the seasonality, s.

Constraints:

if $\mathbf{ds} > 0$, $\mathbf{s} > 0$;
if $\mathbf{ds} = 0$, $\mathbf{s} \geq 0$.

6: **xd**[**nx**] – double *Output*

On exit: the differenced values in elements 1 to **nxd**, and reconstitution data in the remainder of the array.

7: **nxd** – Integer * *Output*

On exit: the number of differenced values in the array **xd**.

8: **fail** – NagError * *Input/Output*

The NAG error parameter (see the Essential Introduction).

6 Error Indicators and Warnings

NE_INT

On entry, $\mathbf{s} = \langle value \rangle$.
Constraint: $\mathbf{s} \geq 0$.

On entry, $\mathbf{ds} = \langle value \rangle$.
Constraint: $\mathbf{ds} \geq 0$.

On entry, $\mathbf{d} = \langle value \rangle$.
Constraint: $\mathbf{d} \geq 0$.

NE_INT_2

On entry, $\mathbf{s} = 0$ and $\mathbf{ds} > 0$: $\mathbf{ds} = \langle value \rangle$.

NE_INT_4

On entry, $\mathbf{nx} \leq \mathbf{d} + (\mathbf{ds} \times \mathbf{s})$: $\mathbf{nx} = \langle value \rangle$, $\mathbf{d} = \langle value \rangle$ $\mathbf{ds} = \langle value \rangle$, $\mathbf{s} = \langle value \rangle$.

NE_BAD_PARAM

On entry, parameter $\langle value \rangle$ had an illegal value.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please consult NAG for assistance.

7 Accuracy

The computations are believed to be stable.

8 Further Comments

The time taken by the routine is approximately proportional to $(\mathbf{d} + \mathbf{ds}) \times \mathbf{nx}$.

9 Example

The example program reads in a set of data consisting of 20 observations from a time series. Non-seasonal differencing of order 2 and seasonal differencing of order 1 (with seasonality of 4) are applied to the input data, giving an output array holding 14 differenced values and 6 values which can be used to reconstitute the output array.

9.1 Program Text

```
/* nag_tsa_diff (g13aac) Example Program.
 *
 * Copyright 2002 Numerical Algorithms Group.
 *
 * Mark 7, 2002.
 */

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagg13.h>

int main(void)
{
  /* Scalars */
  Integer exit_status, i, d, ds, s, nx, nxd;
  NagError fail;

  /* Arrays */
  double *x = 0, *xd = 0;

  INIT_FAIL(fail);
  exit_status = 0;
  Vprintf("g13aac Example Program Results\n");

  /* Skip heading in data file */
  Vscanf("%*[^\n] ");

  Vscanf("%ld%ld%ld%ld%*[^\n] ", &nx, &d, &ds, &s);

  if (nx > 0)
    {
      /* Allocate memory */
      if ( !(x = NAG_ALLOC(nx, double)) ||
           !(xd = NAG_ALLOC(nx, double)) )
        {
          Vprintf("Allocation failure\n");
          exit_status = -1;
          goto END;
        }

      for (i = 1; i <= nx; ++i)
        Vscanf("%lf", &x[i-1]);
      Vscanf("%*[^\n] ");

      Vprintf("\n");
      Vprintf("Non-seasonal differencing of order %ld "
              "and seasonal differencing\nof order %ld "
              "with seasonality %ld are applied\n", d, ds, s);
```

```
      g13aac(x, nx, d, ds, s, xd, &nxd, &fail);
      if (fail.code != NE_NOERROR)
        {
          Vprintf("Error from g13aac.\n%s\n",   fail.message);
          exit_status = 1;
          goto END;
        }

      Vprintf("\n");
      Vprintf("The output array holds %2ld values, of which the "
              "first %2ld are differenced values\n\n", nx, nxd);

      for (i = 1; i <= nx; ++i)
        {
          Vprintf("%10.1f", xd[i-1]);
          if (i % 5 == 0 || i == nx)
            Vprintf("\n");
        }
    }

 END:
  if (x) NAG_FREE(x);
  if (xd) NAG_FREE(xd);

  return exit_status;
}
```

9.2 Program Data

```
g13aac Example Program Data
 20  2  1  4
 120.0 108.0  98.0 118.0 135.0
 131.0 118.0 125.0 121.0 100.0
  82.0  82.0  89.0  88.0  86.0
  96.0 108.0 110.0  99.0 105.0
```

9.3 Program Results

```
g13aac Example Program Results

Non-seasonal differencing of order 2 and seasonal differencing
of order 1 with seasonality 4 are applied

The output array holds 20 values, of which the first 14 are differenced values

     -11.0     -10.0      -8.0       4.0      12.0
      -2.0      18.0       9.0      -4.0      -6.0
      -5.0      -2.0     -12.0       5.0       2.0
     -10.0     -13.0      17.0       6.0     105.0
```